2.26

Situation:

Viscosity of SAE 10W-30 oil, kerosene and water.

$T = 38\,^\circ\text{C} = 100\,^\circ\text{F}$.

Find:

Dynamic and kinematic viscosity of each fluid.

PLAN

Use property data found in Table A.4, Fig. A.2 and Table A.5.

SOLUTION

	Oil (SAE 10W-30)	kerosene		water
$\mu(\text{N}\cdot\text{s/m}^2)$	6.7×10^{-2}	1.4×10^{-3}	(Fig. A-2)	6.8×10^{-4}
$\rho(\text{kg/m}^3)$	880	814		993
$\nu(\text{m}^2/\text{s})$	7.6×10^{-5}	1.7×10^{-6}	(Fig. A-2)	6.8×10^{-7}

2.33

Situation:

Glycerin is flowing in between two stationary plates. The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)$$

$dp/dx = -1.6\,\mathrm{kN/m^3} - 1.6\,\mathrm{kPa/m}$, $B = 5\,\mathrm{cm}$.

Find:

Velocity and shear stress at a distance of 12 mm from wall (i.e. at $y = 12\,\mathrm{mm}$).
Velocity and shear stress at the wall (i.e. at $y = 0\,\mathrm{mm}$).

Properties:

Glycerin (20 °C), Table A.4: $\mu = 1.41\,\mathrm{N\cdot s/m^2}$.

PLAN

Find velocity by direct substitution into the specified velocity distribution.
Find shear stress using the definition of viscosity: $\tau = \mu\,(du/dy)$, where the rate-of-strain (i.e. the derivative du/dy) is found by differentiating the velocity distribution.

SOLUTION

a.) Velocity (at $y = 12\,\mathrm{mm}$)

$$\begin{aligned}
u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\
&= -\frac{1}{2\,(1.41\,\mathrm{N\cdot s/m^2})}\left(-1600\,\mathrm{N/m^3}\right)\left((0.05\,\mathrm{m})\,(0.012\,\mathrm{m}) - (0.012\,\mathrm{m})^2\right) \\
&= 0.258\,7\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}$$

$$\boxed{u\,(y = 12\,\mathrm{mm}) = 0.259\,\mathrm{m/s}}$$

Rate of strain (general expression)

$$\begin{aligned}
\frac{du}{dy} &= \frac{d}{dy}\left(-\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\right) \\
&= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)\frac{d}{dy}\left(By - y^2\right) \\
&= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y)
\end{aligned}$$

Rate of strain (at $y = 12\,\mathrm{mm}$)

$$\begin{aligned}
\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \\
&= \left(-\frac{1}{2\,(1.41\,\mathrm{N\cdot s/m^2})}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m^3}}\right)(0.05\,\mathrm{m} - 2\times 0.012\,\mathrm{m}) \\
&= 14.75\,\mathrm{s^{-1}}
\end{aligned}$$

Definition of viscosity

$$
\begin{aligned}
\tau &= \mu \frac{du}{dy} \\
&= \left(1.41 \frac{\mathrm{N \cdot s}}{\mathrm{m^2}}\right)\left(14.75\,\mathrm{s^{-1}}\right) \\
&= 20.798\,\mathrm{Pa}
\end{aligned}
$$

$$\boxed{\tau\,(y = 12\,\mathrm{mm}) = 20.8\,\mathrm{Pa}}$$

b.) Velocity (at $y = 0\,\mathrm{mm}$)

$$
\begin{aligned}
u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\
&= -\frac{1}{2\left(1.41\,\mathrm{N \cdot s/m^2}\right)}\left(-1600\,\mathrm{N/m^3}\right)\left((0.05\,\mathrm{m})(0\,\mathrm{m}) - (0\,\mathrm{m})^2\right) \\
&= 0.00\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}
$$

$$\boxed{u\,(y = 0\,\mathrm{mm}) = 0\,\mathrm{m/s}}$$

Rate of strain (at $y = 0\,\mathrm{mm}$)

$$
\begin{aligned}
\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \\
&= \left(-\frac{1}{2\left(1.41\,\mathrm{N \cdot s/m^2}\right)}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m^3}}\right)(0.05\,\mathrm{m} - 2 \times 0\,\mathrm{m}) \\
&= 28.37\,\mathrm{s^{-1}}
\end{aligned}
$$

Shear stress (at $y = 0\,\mathrm{mm}$)

$$
\begin{aligned}
\tau &= \mu \frac{du}{dy} \\
&= \left(1.41 \frac{\mathrm{N \cdot s}}{\mathrm{m^2}}\right)\left(28.37\,\mathrm{s^{-1}}\right) \\
&= 40.00\,\mathrm{Pa}
\end{aligned}
$$

$$\boxed{\tau\,(y = 0\,\mathrm{mm}) = 40.0\,\mathrm{Pa}}$$

REVIEW

1. As expected, the velocity at the wall (i.e. at $y = 0$) is zero due to the no slip condition.
2. As expected, the shear stress at the wall is larger than the shear stress away from the wall. This is because shear stress is maximum at the wall and zero along the centerline (i.e. at $y = B/2$).

2.59

Situation:

A water bug is balanced on the surface of a water pond.
$n = 6$ legs, $\ell = 5\,\text{mm/leg}$.

Find:

Maximum mass of bug to avoid sinking.

Properties:

Surface tension of water, from Table A.4, $\sigma = 0.073$ N/m.

PLAN

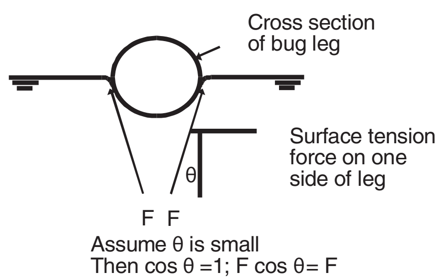
Apply equilibrium, then the surface tension force equation.

SOLUTION

Force equilibrium

$$\begin{aligned} \text{Upward force due to surface tension} &= \text{Weight of Bug} \\ F_T &= mg \end{aligned}$$

To find the force of surface tension (F_T), consider the cross section of one leg of the bug:



Surface tension force

$$\begin{aligned} F_T &= (2/\text{leg})(6 \text{ legs})\sigma\ell \\ &= 12\sigma\ell \\ &= 12(0.073 \text{ N/m})(0.005 \text{ m}) \\ &= 0.00438\,\text{N} \end{aligned}$$

Apply equilibrium

$$\begin{aligned} F_T - mg &= 0 \\ m &= \frac{F_T}{g} = \frac{0.00438\,\text{N}}{9.81\,\text{m}^2/\,\text{s}} \\ &= 0.4465 \times 10^{-3}\,\text{kg} \end{aligned}$$

$$\boxed{m = 0.447\,\text{g}}$$

2.60

Situation:

A water column in a glass tube is used to measure pressure.

$d_1 = 6.35\,\text{mm}$, $d_2 = 3.2\,\text{mm}$, $d_3 = 0.8\,\text{mm}$.

Find:

Height of water column due to surface tension effects for all diameters.

Assumptions:

Assume that $\theta = 0$.

Properties:

From Table A.4: surface tension of water is $0.073\,\text{N/m}$

SOLUTION

Surface tension force

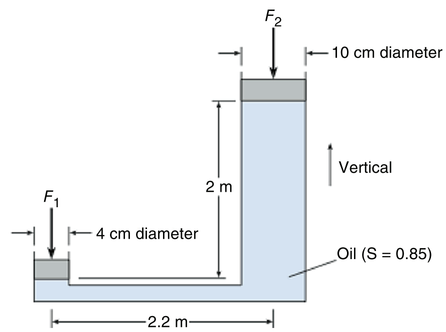
$$\begin{aligned}
\Delta h &= \frac{4\sigma}{\gamma d} = \frac{0.073\,\text{N/m}}{9800\,\text{N/m}^3 \times d\text{ m}} = \frac{2.98\times10^{-5}\,\text{m}^2}{d\text{ m}} \\
d &= 6.35\text{ mm} = 0.0635\text{ m};\ \Delta h = \frac{2.98\times10^{-5}\,\text{m}^2}{0.00635\,\text{m}} = 0.00469\,\text{m} = \boxed{4.69\text{ mm}} \\
d &= 3.2\text{ mm} = 0.0032\text{ m};\ \Delta h = \frac{2.98\times10^{-5}\,\text{m}^2}{0.0032\,\text{m}} = 0.00931\,\text{m} = \boxed{9.31\text{ mm}} \\
d &= 0.8\text{ mm} = 0.0008\text{ m};\ \Delta h = \frac{2.98\times10^{-5}\,\text{m}^2}{0.0008\,\text{m}} = 0.0373\,\text{m} = \boxed{37.3\text{ mm}}
\end{aligned}$$

3.17: PROBLEM DEFINITION

Situation:

A force is applied to a piston.

$F_1 = 200\,\text{N}$, $d_1 = 4\,\text{cm}$, $d_2 = 10\,\text{cm}$.



Find:

Force resisted by piston.

Assumptions:

Neglect piston weight.

PLAN

Apply the hydrostatic equation and equilibrium.

SOLUTION

1. Equilibrium (piston 1)

$$\begin{aligned} F_1 &= p_1 A_1 \\ p_1 &= \frac{F_1}{A_1} \\ &= \frac{4 \times 200\,\text{N}}{\pi \cdot (0.04\,\text{m})^2\ \text{m}^2} \\ &= 1.59 \times 10^5\,\text{Pa} \end{aligned}$$

2. Hydrostatic equation

$$\begin{aligned} p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\ p_2 &= p_1 + (S\gamma_{\text{water}})(z_1 - z_2) \\ &= 1.59 \times 10^5\,\text{Pa} + \left(0.85 \times 9810\,\text{N}/\,\text{m}^3\right)(-2\,\text{m}) \\ &= 1.423 \times 10^5\,\text{Pa} \end{aligned}$$

3. Equilibrium (piston 2)

$$
\begin{aligned}
F_2 &= p_2 A_2 \\
&= \left(1.425 \times 10^5 \,\mathrm{N/\,m^2}\right)\left(\frac{\pi\,(0.1\,\mathrm{m})^2}{4}\right) \\
&= 1119\,\mathrm{N}
\end{aligned}
$$

$$
\boxed{F_2 = 1120 \text{ N}}
$$

3.19: PROBLEM DEFINITION

Situation:
A diver goes underwater.
$\Delta z = 50\,\text{m}$.

Find:
Gage pressure (kPa).
Ratio of pressure to normal atmospheric pressure.

Properties:
Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

1. Apply the hydrostatic equation.
2. Calculate the pressure ratio (use absolute pressure values).

SOLUTION

1. Hydrostatic equation

$$\begin{aligned} p &= \gamma \Delta z = 9790\,\text{N/m}^3 \times 50\,\text{m} \\ &= 489,500\ \text{N/m}^2 \end{aligned}$$

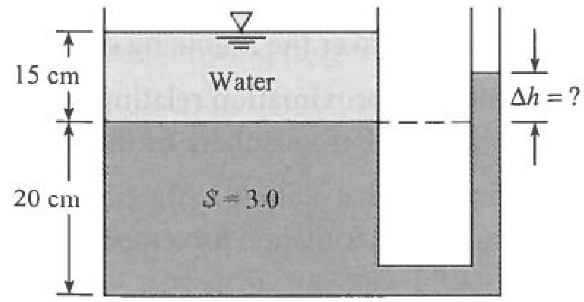
$$\boxed{p = 490\ \text{kPa gage}}$$

2. Calculate pressure ratio

$$\frac{p_{50}}{p_{\text{atm}}} = \frac{489.5\,\text{kPa} + 101.3\,\text{kPa}}{101.3\,\text{kPa}}$$

$$\boxed{\frac{p_{50}}{p_{\text{atm}}} = 5.83}$$

3.23



PROBLEM 3.23

1. Manometer equation:

$$p_1 + \gamma_{water} h_{water} + \gamma_{liquid} h_{liquid} = p_2 + \gamma_{liquid}(h_{liquid} + \Delta h)$$

2. Given conditions:

$$p_1 = p_2 = p_{atm} \text{ (Both ends are open to the atmosphere)}$$

$$\gamma_{liquid} = S\gamma_{water} = 3.0\, \gamma_{water}$$

$$h_{water} = 15\ cm, \quad h_{liquid} = 20\ cm$$

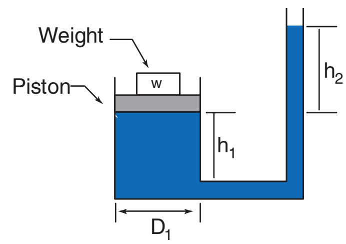
3. Substitute and solve:

$$\Delta h = h_{water}/3 = 5 \text{ cm}$$

3.24: PROBLEM DEFINITION

Situation:

A mass sits on top of a piston situated above a reservoir of oil.



Find:

Derive an equation for h_2 in terms of the specified parameters.

Assumptions:

Neglect the mass of the piston.
Neglect friction between the piston and the cylinder wall.
The pressure at the top of the oil column is 0 kPa-gage.

PLAN

1. Relate w to pressure acting on the bottom of the piston using equilibrium.
2. Related pressure on the bottom of the piston to the oil column height using the hydrostatic equation.
3. Find h_2 by combining steps 1 and 2.

SOLUTION

1. Equilibrium (piston):

$$w = p_1 \left(\frac{\pi D_1^2}{4} \right) \tag{1}$$

2. Hydrostatic equation. (point 1 at btm of piston; point 2 at top of oil column):

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{p_1}{S\gamma_{\text{water}}} + 0 &= 0 + h_2 \\ p_1 &= S\ \gamma_{\text{water}}\ h_2 \end{aligned} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$mg = S\ \gamma_{\text{water}}\ h_2 \left(\frac{\pi D_1^2}{4} \right)$$

Answer: $$h_2 = \frac{4w}{(S)(\gamma_{\text{water}})(\pi D_1^2)}$$

REVIEW

1. Notice. Column height h_2 increases linearly with increasing weight w. Similarly, h_2 decreases linearly with S and decreases quadratically with D_1.
2. Notice. The apparatus involved in the problem could be used to create an instrument for weighing an object.